THE CONCEPT OF FORCE: AN EDUCATIONAL ITINERARY WITH A HISTORICAL SUPPORT

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Abstract

The problem here dealt with concerns physics education, and specifically the concept of force. The idea behind this research is that a historical approach to the teaching of such a notion is of great help for the students to fully understand the meaning of this basic physical magnitude. For, most of scientific concepts can be better grasped by the learners if their genetic phase is presented and the problems that induced scientists to introduce them are expounded. In this way, the students will catch the rationale behind the birth of scientific notions. Furthermore, a shared educational itinerary among the teachers of physics, mathematics, history, and philosophy can be conceived so as to realize an interdisciplinary approach to science education. It is often presented as a desideratum, but is rarely concretely developed. Here, restricting to the notion of force, such an approach is developed. Since force is strictly connected to inertia and mass, a large part of this discussion is dedicated to such two notions.

Keywords. *Newton, physics education, history of science, mass, inertia, force, principles of dynamics, inertial reference frames, non-inertial reference frames*

Introduction

The concept of interdisciplinary instruction is one of the basic notions of modern education. Nevertheless, there are not many cases in which genuine interdisciplinary educational itineraries are proposed. In this research, I will present one: the final purpose is to highlight the main characteristics of the concept of force in physics. Force is strictly connected with mass and inertia, so that it is impossible to deal with the first magnitude without also addressing the nature of the other two. Thence, the protagonists of this writing are mass, inertia, and force.

In many cases, even though not in all cases, the best approach for the students to deeply understand scientific concepts is the historical one. For they will be able to grasp the nature of the concepts in their nascent phase, and this shall allow them to catch all the difficulties that lay behind the birth of some notions, which is sometimes obscure if a more formal approach is proposed. The further advantage of the historical method is that it permits to understand the evolution of science and the relationship between science and the phenomena that science itself attempts to model. Indeed, throughout the history of science, there were considerable difficulties in distinguishing between what belongs to the world and what, on the other hand, belongs to our model of the world. The analysis of these difficulties is a useful tool in the hands of the teacher to help students understand the nature of science.

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Therefore, an interdisciplinary educational approach in which history of science represents a support for science education is here proposed (for the importance of history of physics in physics education see, e.g., Bussotti, 2021, Bussotti, 2023b, Galili, 1993, 2001, Galili &Bar, 1992, Lopes Coelho, 2007, 2010, 2012, 2018, Tomara, Tselfes & Gouscos, 2017. For the relation between mathematics and history of mathematics, see Bussotti (2012, 2013, 2014, 2015b, 2019, 2023a). In what follows, several considerations which also have a philosophical value are presented. A typical example is the just mentioned problem regarding the relations between our scientific models and the reality they describe.

In Italy, force is introduced in the third year of high school, when the learners are between 16 and 17 years old. In the lyceums, history and philosophy are also taught, so that an educational project as the proposed one can be performed. This is, obviously, the case in all schools where physics, history, and philosophy are taught. If, as in many countries happens, philosophy is not taught, the considerations here presented can be developed by a collaboration between the teacher of physics and the one of history.

While speaking of mass and force, there is no doubt the fundamental reference text is the monumental work by Isaac Newton (1642-1727) *Philosophiae naturalis principia mathematica* (*Mathematical principles of natural philosophy*, Newton, 1687, 1713, 1726, resp. first, second and third edition. English translation, Newton 1999). With reference to inertia, Galileo Galilei (1564-1642), René Descartes (1596-1650) and Christiaan Huygens (1629-1695) grasped many aspects of such fundamental as well as elusive concept, but the first scientist who constructed a whole system of physical causality relying on the concept of inertia was Newton. Ergo, in this case too, the paramount reference text will be his masterpiece. It is appropriate to stress that we will restrict to dynamics and to classical physics. Therefore, there will be no references to the notion of force in relativity and in quantum mechanics.

The educational considerations here developed are dedicated to the pupils attending the last three years of the high school, when mechanics is dealt with. In this research, the reader will also find some considerations which are more complex and cannot be proposed to the students. They are dedicated to the teachers for them to contextualize the concept of force in a more profound manner. The main aim of this research is to show the way in which history of science can be used in science education. The method here followed consists in referring to the ideas developed by Newton, the father of modern physics, on the concept of force, to comment them and to see how they can be applied in an educational context.

The Concept of Mass

This notion is a cornerstone of modern physics. Before a specific lesson dedicated to it, almost surely, the students do not separate mass from weight. This is comprehensible: in the course of the history, great scientists did not distinguish between the two notions, and the ideas were confused until Newton gave the first definition of mass. At the beginning of his masterpiece, as the first definition, he wrote:

> *Quantity of matter is a measure of matter that arises from its density and volume jointly.*
> If the density of air is doubled in a space that is also doubled, there is four times as much

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> air, and there is six times as much if the space is tripled. The case is the same for snow and powders condensed by compression or liquefaction, and also for all bodies that are condensed in various ways by any causes whatsoever (Newton, 1999, p. 403).

Thus, Newton defines mass as the quantity of matter. Since he was an atomist, mass is given by the quantity of atoms in a given volume. It is appropriate to recall to students that this definition is not satisfactory under one respect: it involves the notion of density, which is not easy and cannot be given for granted. For there is the risk of circularity: to define mass through density and density through mass, which obviously does not work from a logical standpoint.

However, Newton's merit is enormous: he identifies mass as an intrinsic property of each body, which is not the case for weight, which is, instead, equal to mass multiplied by gravity acceleration. It is to point out that here it is not necessary to introduce the concept of force. The notion of acceleration is enough. It is appropriate to anticipate to the students that the origin of gravity acceleration will be clarified after the introduction of the concept of force and that, for the moment, the notion of acceleration is sufficient.

This means that, given a body, its mass is the same everywhere, whereas its weight varies: on the Earth, gravity acceleration has a value; on Jupiter, a bigger value; on the Moon, a smaller value, and in the void space its value is null. The weight varies consequently, whereas the mass does not.

The second definition concerns the quantity of motion as the product or mass by velocity. The third is fundamental and sounds like this:

> *Inherent force of matter is the power of resisting by which every body, as far as it is able, perseveres in its state either of resting or of moving uniformly straight forward.* This force is always proportional to the body and does not differ in any way from the inertia of the mass except in the manner in which it is conceived. Because of the inertia of matter, every body is only with difficulty put out of its state, either of resting or of moving. Consequently, inherent force may also be called by the very significant name of force of inertia. (Newton 1999, p. 404).

Here, there is an anticipation of the inertia principle with which we will deal in the next section. In this context, the aspect to be remarked is that here mass is not associated to weight, but to another property of the bodies: their power to resist to any change of their kinetic state. Let us call the mass associated to weight "gravitational mass" and the mass associated to the inherent force "inertial mass". Inherent force has nothing to do with what we call Newtonian force. For the former is an intrinsic property of bodies, whereas the latter indicates an interaction, as we will see. For the moment, let us focus on a further aspect: a physical quantity makes sense if it is possible to measure it. How do we proceed in the case of mass? Suppose we have a trolley at rest; we impart a push to it by machinery. Suppose the road is perfectly smooth, without any friction. The trolley will start with a certain speed. Now, suppose we have another trolley. We use the same machine to push it, but its speed from 0 becomes three times that of the previous trolley. Based on the definition of inertial force, it is easy to extrapolate that the mass of the second trolley is 1/3 of that of the first one. This is, however, certainly not a very practical way of determining masses and, in fact, one proceeds otherwise. We simply use a balance and divide the result by gravity acceleration.

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Nonetheless, we are measuring, at least in principle, two different quantities: for, the first way of measuring mass has nothing to do with Earth's gravity. It works in every region of the universe. While for the second one, gravity is indispensable. If, in fact, the Earth did not attract bodies, it would be impossible to weigh anything. Surprisingly, the two results show that the two masses are equal. This is astonishing because, in fact, they depend on two very different properties. With the trolley experiment we measure the amount of inertia a body has, its resistance to change of motion, which is why we speak of inertial mass. By weighing a body, we measure the degree to which it is attracted by the Earth. That is why we speak of gravitational mass. That the two masses are equal is not a consequence of the theory at all. It is a mere coincidence. From the point of view of classical physics, the identity of the two masses is accidental.

This is an aspect that the teacher has to underline because such a strange and, at the same time, fundamental coincidence must induce to reflect on the fact that a scientific theory cannot explain every aspect of the phenomena. Newtonian physics, which is so efficient to foresee and explain many phenomena, is not suitable to explain the coincidence between inertial and gravitational mass. One might assume that such a coincidence exists independently of any explanation and that it is inscribed in the nature. As a matter of fact, this is the behaviour held until the theory of general relativity, where it is possible to prove that such coincidence is not casual. Connected with this aspect, there is also a methodological-philosophical one: a scientific theory is never a final view on reality. Science is a continuous approximation to reality, which is destined to catch even more inclusive aspects of reality itself, but that can never catch all of them. This is an idea claimed by many scientists, mathematicians, and philosophers. The first who clearly explained it was Federigo Enriques (Enriques 1906). Newtonian physics is a good approximation of reality; relativity theory is a better one that enables us to catch more profound aspects of reality, such as the coincidence between inertial and gravitational mass.

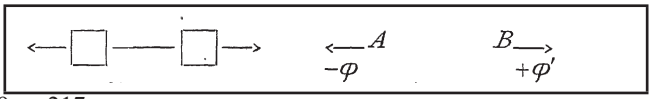
Let us now come back to the definition of quantity of matter given by Newton. We have seen that, although it catches the fundamental difference between mass and weight, it is not completely satisfactory because it relies on the problematic concept of density. To overcome this difficulty, Ernst Mach (1838-1916) gave a relational definition of mass by first defining when two masses are equal. He wrote:

> All those bodies are bodies of equal mass, which, mutually acting on each other, produce in each other equal and opposite accelerations (Mach, 1919, p. 218. See also Barton (1946, Brown (1960, Jammer (1961, Jammer (1999, Roche (2005).

Afterwards, Mach explains as follows: when two bodies have different masses (Figure 1):

Figure 1

The Diagram Used by Mach to Explain His Conception of Mass



Note. Mach 1919, p. 217.

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Consider two bodies *A* and *B*, which, because of their mutual interaction, receive respectively the accelerations -φ and +φ' (Figure 1). In this case, by definition, *B* has φ/φ' times the mass of *A*. This definition of mass has an operative value. For, if one assumes that a body *P* has a unitary mass, then a body *P*' has mass *m* if the acceleration it imparts to *P* is *m* times the acceleration *P* imparts to *P*'. This operative definition depends, hence, on the possibility to measure accelerations, but it is deprived of all the problems deriving from the notions of density, and no theoretical notions are necessary if not kinematical elementary ones such as acceleration. Mach stresses this aspect:

> In our concept of mass, no theory is involved; "quantity of matter" is wholly unnecessary in it; all it contains is the exact establishment, designation, and denomination of a fact (Mach, 1919, p. 218).

A problem could arise: is it true that two bodies which, acting with *A* behave as equal masses, also act as equal masses in a reciprocal interaction? Mach underlines that this cannot be given for granted because there is no logical necessity that imposes this to be the case. Through a physical reasoning, Mach is, anyway, able to show that this happens (Mach, 1919, pp. 219-220).

With Mach's definition, the concept of inertial mass receives a more satisfactory characterization, although, obviously, the problem of the coincidence between inertial and gravitational mass remains unsolved.

From an educational point of view, it is appropriate to highlight the kind of definition used by Mach. It is a definition by abstraction: this means that a concept is defined by indicating when two quantities are equal with respect to that concept. Namely, a class of equivalence is enucleated. Of course, it is then necessary to clarify when two quantities are not equal with respect to that concept. This kind of definition is very important in mathematics and, especially, in modern mathematics. Euclid (IV-III century B.C.) defines the concept of proportion by abstraction (Euclid, 2008, Book V, pp. 130-131), Georg Cantor (1845-1918) defines the concept of cardinal number of a set by abstraction (Cantor, 1878), Gottlob Frege (1848-1925) defines the concept of natural number by abstraction (Frege, 1980, original 1884). These are only significant examples of definitions by abstraction. In the prospective of an interdisciplinary education, these aspects which join physics and abstract mathematics must be underlined. Mathematics is not only the unavoidable instrument of physics; the two disciplines also share basic conceptual elements, such as the possible way of defining the concepts.

The Concept of Inertia

The concept of inertia is the milestone of the whole classical physics. It is difficult, not intuitive, and very abstract. In a serious approach to physics, it is necessary that the learners fully grasp the features of such notion. It is appropriate to deal with inertia in two phases: the first one before introducing the concept of force, which I will do in this section, and the second one after having introduced the concept of force. Since the latter shall be presented in the fourth section, the fifth one will be devoted to the definitive re-conceptualization of inertia.

Although Galilei (Galilei, 1967, II, p. 190; Galilei, 1914, IV, p. 244. See also Bussotti & Pisano, 2025, where references to literature are also referred to), Descartes (Descartes, 1984, pp. 240-241. See also Bussotti & Lotti, 2022, pp. 126-132), and

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Huygens (Huygens, 1703, posthumous. See also Bussotti & Lotti, 2022, pp. 162-177, Stan, 2016) expounded propositions similar to inertia principle and the last one also developed a theory of impacts partially based on it, the first complete physical system based on inertia principle was that of Newton. He stated as follows such a principle or law:

> *Every body perseveres in its state of being at rest or of moving uniformly straight forward, except insofar as it is compelled to change its state by forces impressed.* Projectiles persevere in their motions, except insofar as they are retarded by the resistance of the air and are impelled downward by the force of gravity. A spinning hoop, which has parts that by their cohesion continually draw one another back from rectilinear motions, does not cease to rotate, except insofar as it is retarded by the air. And larger bodies–planets and comets–preserve for a longer time both their progressive and their circular motions, which take place in spaces having less resistance (Newton 1999, p. 416).

To explain this principle, a formal definition of force is not necessary, it is sufficient to think intuitively of an action tending to change the kinetic state of a body. A reappraisal of the principle will be done after the introduction of the concept of force.

Therefore: the state of rest is considered dynamically equivalent to that of uniform rectilinear motion. Both would be eternal, if no external action were present.

First conceptual and educational problem: most of the young people aged 16-17 years are influenced by their perceptive experiences, which seem to be in contradiction with the principle. For if an object is moving on a plane and you apply no external action, it perseveres its motion for a more or less long time, but at a certain instant it stops depending on the material of the plane and of the object. Its motion depends on the form of the object too. It is not necessarily uniform and rectilinear. If one looks at the planetary movements, the young will be impressed by the regularities of such movements, which, certainly, are not rectilinear and uniform, although, at all appearances, no action is exerted on the planets. Let us imagine that a student be particularly gifted and that she/he be able to overcome all these difficulties, also relying upon the explanations of the teacher. A crucial difficulty remains. For this student can ask: "Dear teacher, you speak of rest and uniform motion in a right line, but in respect to what do you determine these kinetic states? For me, in my car, I am at rest and the external world is moving; for the external world, I am moving. What is the situation?" (Bussotti, 2021, p. 4).

The first concept to be explained is that of *relativity of motion*: no body is at rest or in motion in an absolute sense. We are free to choose what body is appropriate to consider at rest based on the problem we have to study: generally, for questions connected to local motion, the Earth is considered at rest, although it is not.

There are several sensorial and visual experiences that can indicate the relativity of motion. Everyone has noted that if a train starts slowly, the passengers have the impression that the train remains at rest and that the station is moving.

Let us now move to the concept of inertia. In the experimental physics laboratory, the teacher could develop such an experiment divided into three phases. In the first phase (Figure 2 left side), two joined equal inclined planes are considered. Be the planes are completely smooth. A ball in A is at the altitude h. Under these conditions, the learners will recognise that, after having reached the basis of the two planes in B, the ball will re-ascend in C at the height h. If you like, you can replace the two inclined planes by a circumference.

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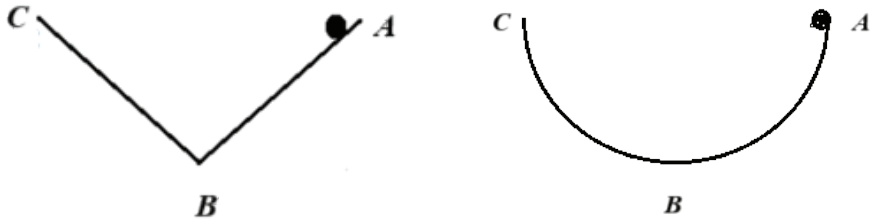
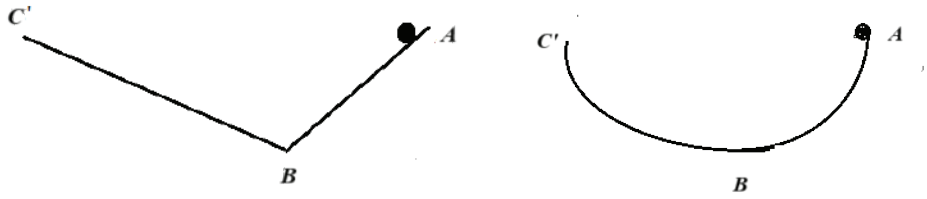
Figure 2
Diagrams Showing the First Part of the Experiment



Note. For the Image on the Left. See Bussotti, 2021, p. 5.

Let us now suppose that the second plane is less inclined on the horizon than *AB*, or, equivalently, that a line having a lesser curvature than the circumference *ABC* is joined in *B* at the quart of circumference *AB* (Figure 3). As before, let the ball start in *A* from the altitude *h*. The students will see that the ball will reach the point *C*', whose height is *h*.

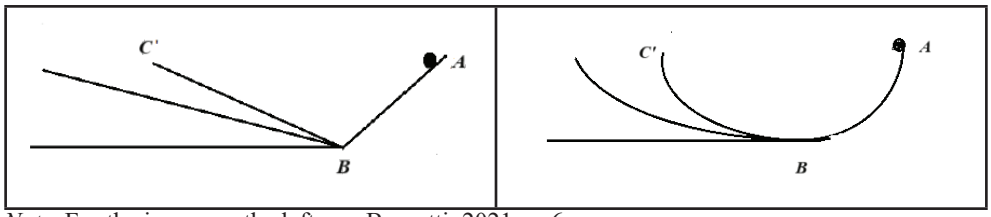
Figure 3
Diagrams Showing the Second Part of the Experiment



Note. For the image on the left, see Bussotti, 2021, p. 5.

Suppose now that the plane *BC*' becomes less and less inclined on the horizon or, equivalently, that the curvature of the line *BC*' continually becomes lesser until being a straight line (Figure 4).

Figure 4
Diagrams Showing the Third Part of the Experiment



Note. For the image on the left, see Bussotti, 2021, p. 6.

https://doi.org/10.33225/BalticSTE/2025.47

Now the teacher can ask his students: suppose the second plane is horizontal; the ball reaches *B*, what happens afterwards? Probably most learners will correctly answer: the ball arrives at *B* with a certain speed, after that it continues its motion along the horizontal plane with the same speed as in *B*.

This simple experiment can, thus, be useful to convince the students that inertia principle holds. Obviously, this is not a proof, because inertia law is a principle, not a theorem. However, it is an experiment which makes the principle intuitively plausible. It is appropriate to recall that Galileo, in the Second Day of his *Dialogue Concerning the Two Chief World Systems* (1632, Galilei, 1967), developed a reasoning whose synthesis has here been explained. The teacher might read in the classroom this passage of the *Dialogue* (*ibidem*, pp. 145-146). This reading is useful to catch all the difficulties that the most brilliant minds encountered to reach inertia law. Thus, the human aspect of science will be highlighted: the scientists reached the notions which are the bases of our knowledge of the external world through a road full of obstacles. Only their hard work permitted to overcome the obstacles (Bussotti, 2021, p. 6).

Some didactical considerations are appropriate: the argumentation here expounded depends on the assumption that the two inclined planes have perfectly smooth surfaces, so that no friction exists. However, this condition is impossible. A surface can have a negligible friction, but not a null friction. What is the consequence of this observation? Is the inertia principle unacceptable from a physical standpoint? The teacher has here an important task: she/he must explain the difference between the universe and our model of the universe to her/his pupils. Inertia principle is an element of our world model, which shows a tendency of the real bodies. It is unavoidable to grasp their kinetic behaviour. In the real universe, no motion is perfectly inertial. The model must provide us with a conceptual scheme such that, if we abstract from some properties *P* (in this case that no surface completely deprived of friction or no body on which no action is exerted can exist), then we can focus on properties *Q*, like inertia, that, although not rigorously existing, can help us to understand profound aspects of the universe. Each situation physics examines deals with our theoretical model, which is a simplified view of reality. Our model is the more satisfactory the more elements of reality it can explain and foresee, but every model will abstract from some aspects of reality, and, hence, it is, for some aspects, incomplete.

A crucial consideration concerns the uniformity and rectilinearity of a motion: we have underlined that every motion is relative. On the other hand, inertia principle seems to speak of uniform rectilinear motion as an absolute kinetic state. How can these two conditions be conciliated? From an operational perspective, this problem can easily be solved. For every problem we address, there is a set of motions which can be regarded as rectilinear and uniform. Let us suppose we want to study the movement of a vehicle on a road without curves and located at a uniform altitude above sea level. If this motion is not accelerated in modulus, it can be regarded as rectilinear and uniform, although it is not because the Earth is not flat, because it is rotating around its axis and around the Sun, and so on. However, if we are interested in the motion of our vehicle, it can be considered rectilinear and uniform when its velocity is subject to no variation in modulus. Therefore, from an operational point of view, there is no problem. Nonetheless, there are many problems from a foundational point of view, namely, if we claim that inertia is an absolute and not an operative concept. We will outline this difficult issue after having introduced the concepts of force and of inertial system.

https://doi.org/10.33225/BalticSTE/2025.47

Finally, a brief observation: according to inertia principle, uniform rectilinear motion continues without the need of any external action. Therefore, this means that such an action does not cause velocities, but changes of velocities, namely accelerations. This action is what Newton called *force*.

The Concept of Force

Since not all motions are rectilinear and uniform, an action is necessary. Let us call such an action force. From an educational perspective, one can choose to simply define force $\boldsymbol{F}$ as the product of mass m by acceleration $\boldsymbol{a}$, so reaching the well known formula $\boldsymbol{F}=m\boldsymbol{a}$. This approach is not wrong but hides several interesting aspects of the concept of force, which can, instead, be highlighted by following Newton's order of ideas, which is what we will do here.

Newton's first Definition concerning force as action is the fourth one of the *Principia*:

> Impressed force is the action exerted on a body to change its state, either of resting or of moving uniformly straight forward (Newton, 1999, p. 405).

As the most significant examples of impressed forces, Newton speaks of impact, pressure, and centripetal force. Impact and pressure act by direct contact between bodies, but what about centripetal force? Newton clarifies his nature in the following fifth Definition:

> Centripetal force is the force by which bodies are drawn from all sides, are impelled, or in any way tend, toward some point as to a centre (*ibid*., p. 405).

The first and fundamental centripetal force mentioned by Newton is gravity.

Here, several educational as well as conceptual observations are necessary:

1) Gravity has a different character with respect to impact and pressure. For, the latter act directly by contact, whereas gravity does not. When a body descends towards the Earth's surface, there is no contact between the body and the surface. The planets, as Newton will prove, gravitate towards the Sun, but there is no direct contact between the planets and the Sun. This means that gravity acts at a distance. In the course of the history of physics there were, before and after Newton, various attempts to find an ethereal medium which could transmit gravity (see, e.g., Bussotti & Lotti, 2022, chapters 5, 6, 8) and Newton himself was not convinced of the action at a distance, as a famous letter to Richard Bentley (1662-1742) on 25 February 1693 clarifies (Newton, 1756, posthumous). However, no satisfying intermediary medium was found, so that gravity in Newtonian physics is, in fact, an action at a distance. Here, an observation is appropriate which connects the teaching of physics with that of history and of philosophy: Descartes (see Descartes 1984, first edition 1644) had tried to propose a series of mechanisms which explained all the actions simply by impacts and motions. His attempt was reasonable: in the Medieval period and, above all, in the Renaissance, astrology and natural magic was developed (see, e.g., Dawes, 2013, Frank, 2009, Hanegraaff, 2012, Humphreys & Wagner, 2013, Kristeller, 1964, Mebane, 1989, Thorndike, 1923). Such

https://doi.org/10.33225/BalticSTE/2025.47

disciplines claimed to explain many effects which take place on the Earth as due to mysterious influxes of asters and celestial bodies which act at a distance. Descartes and other scientists of the 17th century refused such a conception. Thence, Descartes tried to develop a system where, by supposing the existence of a particular matter he called ether, all the interactions could be explained by direct contact. Descartes did not develop the mathematical part of his system, but, especially Huygens, and in part Leibniz did (Bussotti, 2015a). Therefore, in the perspective of Huygens and Leibniz, Newton's action at a distance was criticised because it seemed to reintroduce mysterious entities which physics had tried to rebel against. Leibniz, particularly, developed a series of argumentations against the action at a distance (see, e.g. Bouquiaux, 2008, Bussotti, 2015a, Cox, 2002, Duchesneau, 2004, Gale, 1988, Garber, 2009, Leibniz, 1690, Leibniz, 1977, Look, 2011, Miller, 1982, Ramati, 1996, Roberts, 2003). However, it is to point out that the physics based on the existence of an ethereal medium did not achieve any results that were remotely comparable to those obtained by Newton. This teaches us that science is not a theory of everything: the origin of gravity was unknown, but Newton was perfectly able to explain its effects and, at least, in a certain phase of science, this is absolutely acceptable. It makes no sense to refuse an almost perfectly working theory because the origin of the concept of force that is at its basis is unknown. Of course, this does not mean that research on such origin must be neglected. The picture here presented is useful for the students to understand many nuances of what a scientific theory is and how it is developed.

2) We have spoken of forces acting at a distance and by contact. It is appropriate to add that, when the studies on electricity and magnetism were developed, in great parts thanks to the works of physicists and mathematicians such as Charles Augustin de Coulomb (1736-1806), Luigi Galvani (1737-1798), Alessandro Volta (1745-1827), Carl Friedrich Gauss (1777-1855), Hans Christian Ørsted (1777-1851), Siméon-Denis Poisson (1781-1840), Michael Faraday (1791-1867), James Clerk Maxwell (1831-1879), Faraday had the enormous merit to introduce the concept of magnetic field in physics. This entity was exactly the one that mediates magnetic force. Therefore, magnetism, and after Maxwell, electromagnetism, studies a force which acts by contact through the intermediary of a field. But, in classical physics, although the notion of gravitational field is used, no intermediary comparable to magnetic, or electromagnetic field, has ever been found. It is to add that partially connected to the history of magnetic and electromagnetic fields, there is also that concerning light transmission. Here things are really complicated because, along the whole 19th century, there was a huge discussion on the mysterious intermediary medium (also called ether) which should have transmitted light (Whittaker, 1910).

3) Newton explains masterfully how a curved motion takes place: suppose a body has a certain inertial motion. It arrives close to another body, which is a source of centripetal force. Its motion is deviated and becomes curved. So that a curved motion is due to an initial motion and to the action of a centripetal force. Let us refer to his clear explanation, which is well conceived also from an educational standpoint:

> A stone whirled in a sling endeavors to leave the hand that is whirling it, and by its endeavor it stretches the sling, doing so the more strongly the more swiftly it revolves; and as soon as it is released, it flies away. The force opposed to that endeavor, that is, the

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> force by which the sling continually draws the stone back toward the hand and keeps it in an orbit, I call centripetal, since it is directed toward the hand as toward the center of an orbit. And the same applies to all bodies that are made to move in orbits. They all endeavor to recede from the centers of their orbits, and unless some force opposed to that endeavor is present, restraining them and keeping them in orbits and hence called by me centripetal, they will go off in straight lines with uniform motion. If a projectile were deprived of the force of gravity, it would not be deflected toward the earth but would go off in a straight line into the heavens and do so with uniform motion, provided that the resistance of the air were removed. The projectile, by its gravity, is drawn back from a rectilinear course and continually deflected toward the earth, and this is so to a greater or lesser degree in proportion to its gravity and its velocity of motion. The less its gravity in proportion to its quantity of matter, or the greater the velocity with which it is projected, the less it will deviate from a rectilinear course and the farther it will go (Newton, 1999, pp. 405-406).

After having defined centripetal force, Newton defines its absolute quantity as the intensity of such a force and its accelerative quantity as a magnitude proportional to the acceleration that such a force produces. In an educational context, it is simply appropriate to speak of acceleration.

The last definition, the eighth one, is important because, as a matter of fact, Newton offers the definition $\boldsymbol{F}=m\boldsymbol{a}$:

> The motive quantity of centripetal force is the measure of this force that is proportional to the motion which it generates in a given time (ibidem, p. 407).

There is no doubt that, in this context, Newton meant by motion, the quantity of motion $\boldsymbol{p}$ he had already defined as $\boldsymbol{p}=m\boldsymbol{v}$. He explains the situation as follows:

> Therefore, accelerative force is to motive force as velocity to motion. For quantity of motion arises from velocity and quantity of matter jointly, and motive force from accelerative force and quantity of matter jointly (ibidem, p. 408).

Since the accelerative force is acceleration $\boldsymbol{a}$, the quantity of motion is $m\boldsymbol{v}$, if we indicate the motive force by $\boldsymbol{F}_{\mathrm{m}}$, the proportion follows

$$\boldsymbol{a}:\boldsymbol{F}_{\mathrm{m}}=\boldsymbol{v}:m\boldsymbol{v}.$$

Therefore, $\boldsymbol{F}_{\mathrm{m}}=m\boldsymbol{a}$. Newton was restricted to considering centripetal forces because he was particularly interested in gravity. Nonetheless, there is no problem to offer such a definition as that general for a force. However, after the whole educational itinerary here presented, this definition has a full justification, the conceptual and historical reasons for its introduction are clear, and this concept does not appear as something imposed without any justification. A brief observation can be added: Newtonian historiography has lengthy dealt with the question whether Newton regarded a continuous force, like gravity, as the sum of an infinite series of infinitesimal impulses (see, e.g. Bernard Cohen, 1967, Blay, 2001, Ellis, 1965, Ellis 1976, Erlichson, 1991, Jammer, 1962, Pourciau, 2006, Westfall, 1971). There is no doubt that, in some circumstances, for example in Proposition I, where he proved the area law, this is the case. However, Newton's procedure is related to his mathematical technique rather than to his physical conception of force, and I support

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the idea that he did not consider a continuous force as generated by impulses. Since this topic is not directly connected to our educational context, it is not addressed to it. The reader can form his own opinion by resorting to Newton's text and to the mentioned literature.

We are now ready to introduce the second law of motion, which sounds:

> A change in motion is proportional to the motive force impressed and takes place along the straight line in which that force is impressed (Newton 1999, p. 416).

Let us now develop some observations:

1) The first one concerns the concept of "change of motion". There are many interpretations on what Newton meant by this expression (the previous reference to literature also applies in this case). This is a fundamental issue for Newtonian historiography. However, considering that in the eighth Definition Newton identified motion with quantity of motion, it seems reasonable to assume that "change of motion" means "change in quantity of motion" (see, e.g., Jammer, 1962, pp. 124-126), that is $\boldsymbol{F} = \frac{d(m\boldsymbol{v})}{dt} = m\frac{d\boldsymbol{v}}{dt} = m\boldsymbol{a}$. This choice is also the easiest from an educational point of view.

2) The second observation regards a crucial aspect: one might get the idea that Newton's second law is a definition of force. But, in fact, it is not. Newton had already defined force, as we have seen, and forces can be measured independently of dynamics. For example, the force exerted by a spring is experimentally found to be proportional to the spring compression or elongation, according to Hooke's law. Therefore, there is no reason to offer a further statement equivalent to a definition. For, the second law establishes an equation between a quantity whose expression is fixed and a quantity whose expression is variable from problem to problem. The former is $m\boldsymbol{a}$, or, as it is usual to write in dynamics $m\ddot{\boldsymbol{x}}$, where the overwritten dot indicates derivative (in this case of position x) with respect to time, and two dots the second derivative with respect to time. The aim is to find the expression of position in a function of time, the so-called equation of motion.

An example which is not so trivial as to be obvious, nor so difficult as to be problematic for the students, is that of a simple pendulum (Figure 5. The treatment is drawn from Sonego 2023, pp. 81-82).

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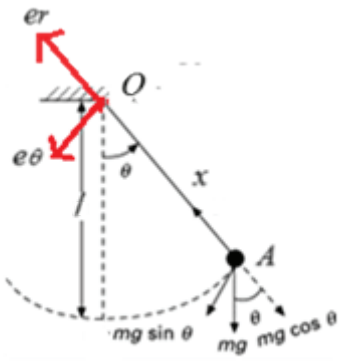
Figure 5
Diagram Describing the Movement of a Simple Pendulum



Suppose no friction exists. Thence, the particle A is subject to its weight $m\boldsymbol{g}$ and to the constraint force $\boldsymbol{\phi}$, which is perpendicular to the circumference on which A moves. Let us introduce an orthonormal basis $(\boldsymbol{e}_r,\boldsymbol{e}_\theta)$, with parallel to $\boldsymbol{x}$, so that $\boldsymbol{x} = \boldsymbol{OP} = l\boldsymbol{e}_r$ and $\boldsymbol{\phi} = \phi\boldsymbol{e}_r$. Therefore, Newton's second law supplies the equation

$$m\ddot{\boldsymbol{x}} = m\boldsymbol{g} + \boldsymbol{\phi} \qquad 1).$$

As we have told, $m\ddot{\boldsymbol{x}}$ is the general fix expression which indicates all the forces acting on a system and $m\boldsymbol{g} + \boldsymbol{\phi}$ indicates the specific forces acting in the problem of the simple pendulum. Looking at the figure, it follows immediately

$$m\ddot{\boldsymbol{x}} \cdot \boldsymbol{e}_\theta = m\boldsymbol{g} \cdot \boldsymbol{e}_\theta = -mg\sin\theta \qquad 2),$$

where the dot indicates scalar product and θ is the angle that $\boldsymbol{OP}$ forms with the downwards direction through O, measured positively in the counterclockwise direction. The velocity of A is $\dot{\boldsymbol{x}} = l\dot{\boldsymbol{e}}_r$. Since, it is easy to prove that $\dot{\boldsymbol{e}}_r = \dot{\theta}\,\boldsymbol{e}_\theta$, it follows that $\dot{\boldsymbol{x}} = \dot{\theta}\,\boldsymbol{e}_\theta$. By differentiating with respect to time, we find the acceleration:

$$\ddot{\boldsymbol{x}} = -l\dot{\theta}^2\boldsymbol{e}_r + l\ddot{\theta}\boldsymbol{e}_\theta.$$

Replacing this value in Equation 2, one finds the differential equation for the function $\theta(t)$:

$$\ddot{\theta} + g/l\sin\theta = 0.$$

If equation 1) is multiplied scalarly by $\boldsymbol{e}_r$ the following equation for the constrain-force is obtained:

$$\phi = -ml\dot{\theta}^2 - mg\cos\theta.$$

https://doi.org/10.33225/BalticSTE/2025.47

Our problem has now to be reduced to the solution of two differential equations, which, in this case, can be solved analytically, so as to offer all the physical quantities required. Of course, in most cases, the equations cannot be solved analytically, but the form and the purposes of Newton's second law are clear: offering the equations of motion.

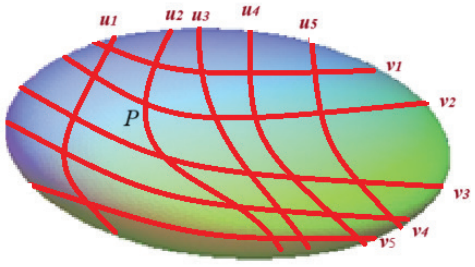
3) In Corollary I to his laws of motions (Newton 1999, pp. 417-418) Newton established the rule for the sum of two vectors (parallelogram rule) as well as the property that the effect of a force on a body does not depend on the body's state of rest or motion or on the presence of other forces simultaneously acting on the body.

4) The following one is an observation of utmost importance from the conceptual and educational standpoint: several scholars, among whom Mach (Mach, 1919, p. 242), claim that the first law is included in the second one or, even, in the definition of force. This is false, and Newton was completely correct to separate the definition of force, the first and second laws. This involves a profound relation: that between inertial and non-inertial reference-frames, which is the topic we will address in the next section.

Inertial and Non-Inertial Reference Frames

It is impossible to develop physics without the notion of a reference frame. This is connected to four aspects: 1) identification of the position of an object, 2) metric questions, 3) kinematical and 4) dynamical issues: with regard to the first two aspects, a system of coordinates is sufficient. As to the position of a point, it is not necessary to have introduced a metric; systems of lines are sufficient. If, for example, we want to identify a point on a bidimensional surface (e.g. an ellipsoid) we can draw two systems of lines u_i and v_i which are subject to only three conditions: 1) each line of both systems does not intersect itself; 2) each line of both systems does not intersect any line of the same system; 3) each line u_i of the system u intersects all the lines v_i of the system v and vice versa (Figure 6). This is sufficient to identify the position of every point on the ellipsoid without any reference to the concept of distance. For example, the point P will have coordinates (u_2, v_2).

Figure 6
A System of Gaussian Coordinates on an Ellipsoid



The merit of having introduced the general concept of this system of coordinates is by Gauss (Gauss, 2005), even though the parallel and meridians are Gaussian coordinates

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and, of course, were used before Gauss. Gaussian coordinates are fundamental in general relativity and can be introduced by the teacher without any particular problem, provided it be clarified that metric is not necessary to identify a point.

If, besides identifying a point, we want also to calculate distances, a metric structure must be introduced in our system of coordinates. At the high school level, it is sufficient to introduce the Euclidean metric for Cartesian and polar/spherical coordinates. This can be done by adding an origin to our system of coordinates and defining, in Cartesian coordinates, the distance of a point $P(x,y)$ from the origin $O(0,0)$ as $\sqrt{(x^2+y^2}$), if one is working in two dimensions, and of a point $Q(x,y,z)$ from the origin $O(0,0,0)$ as $\sqrt{(x^2+y^2+z^2}$, if one is working in space. The distance between two any points of coordinates (x_1,y_1,z_1) and (x_2,y_2,z_2) will, thence, be $\sqrt{(x_1-x_2)^2 + (y_1 - y_2)^2 + (z_1-z_2)^2}$. The distance in polar and spherical coordinates presents no particular problem and can be found in every handbook of mathematics for high schools.

Until now, we have spoken of geometry and systems of coordinates, but we need to make physics. Therefore, we have to introduce the concept of a reference frame. In physics, a reference system is a system of coordinates relative to which a certain physical phenomenon or object is observed and measured, or certain measurements are taken. This notion is necessary in kinematics and dynamics for the description of the motion of bodies and the observation that motion is always relative to a system with respect to which we observe it. This means that, while a system of coordinates in geometry is a static object (this does not mean that geometrical objects are always regarded as static, for the notion of geometrical motion exists), a reference frame can have every kind of motion with respect to another reference frame.

Let us now come to one of the most fundamental concepts of the whole physics: that of the inertial reference frame. An inertial reference frame is characterised by the condition that, if a material point is free, i.e., subjected to a zero resultant of forces, then it will persist in its state of rest or uniform rectilinear motion as long as it is not disturbed. In other words, an observer or reference frame S is said to be inertial if, given an isolated material point of mass P, the acceleration of P in S is equal to zero, whatever the instant t at which this measurement is made and whatever the kinematic state $(P, \dot{P})$ of the point at the same instant t be.

It can be verified that the observers, who measure the acceleration of an isolated point to be null, are all and only those moving with uniform rectilinear translatory motion with respect to the observer S.

This means that an inertial reference frame is one in which the first law of dynamics holds. Therefore, to change the kinetic state of a body in an inertial reference frame, a force is necessary. This is the reason why the second law depends, in fact, on the first law. Namely, $\boldsymbol{F}=m\boldsymbol{a}$ only in inertial reference frames. An example will clarify what is claimed (Figure 7).

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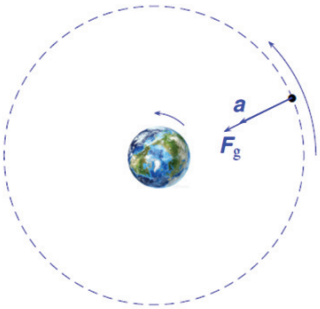
Figure 7

The Movement of a Geostatic Satellite



Let us consider a geostatic satellite. Let us set the origin of our reference frame in the Earth's centre and suppose that the frame does not participate in the diurnal motion of the Earth. Be *M* the mass of the Earth, *m* the mass of the satellite, $F_g = G\frac{mM}{r^2}$ gravity force, where *G* is the Newton constant and *r* represents the distance from the Earth's centre at which the satellite must be set for it to be geostatic, $a = \omega^2 r$ be the satellite's centripetal acceleration, fulfilling the condition $\omega = \frac{2\pi}{T}$, with *T*= 1 day. Gravity force and centripetal force to which the satellite is subject have the same value. Thence, the equation

$$ma = m\omega^2 r = G\frac{mM}{r^2}$$

holds, so that $r = \sqrt[3]{\frac{MG}{\omega^2}}$.

However, no one prevents us from considering a non-inertial reference frame corotating with Earth. In this case, gravity force remains the same, namely $F_g = G\frac{mM}{r^2}$, but in this case, the satellite has no centripetal acceleration *a'* because it is at rest with respect to the centre of the reference frame. This means that *a' = 0*, and, certainly, force is different from ***ma'***.

Therefore, the first Newton's law is necessary, and it establishes the frames for which ***F=ma***, namely the frames in which Newton's second law can be applied. This is a crucial aspect the students must fully understand.

In non-inertial reference frames, other force terms can appear (centrifugal, Coriolis, and Euler forces), but this important development goes beyond the aims of this paper.

Let us now move to another observation which is significant in an educational perspective: reference frames, be they inertial or non-inertial, belong to our model of the physical universe, to our theory. In the world, there is no reference frame. There are events and objects, but not reference frames. The latter are our constructions necessary to develop science. From a historical perspective, the concept of inertial reference frame was formally introduced in 1885 by the German physicist Ludwig Lange (1863-1936) (Barbour & Pfister, 1995, Lange, 1885a, Lange, 1885b, Bussotti & Lotti, 2022, pp. 225-227), but, obviously, concepts analogous to that of inertial frame were already used by

https://doi.org/10.33225/BalticSTE/2025.47

Newton, and even by Galileo. The difference is that Lange clearly distinguished what belongs to our model of the theory from what belongs to the universe. Before Lange, the equivalent of an inertial reference frame was an object or a portion of the physical space sufficiently far from external actions as to be regarded as subject only to an inertial motion. Of course, in the physical world, no object or no portion of space is completely force-free, but there are objects or portion of space which, in relation to the problem one is going to study, can be regarded as subject only to an inertial motion without any detectable error. Thence, the notion of inertial reference frame is precise and belongs to the theory; in the universe, one finds portion of space which can approximately be considered as inertial. The most famous and paradigmatic example is "il gran navilio" ("the big ship") of Galileo: he imagined a ship moving uniformly and rectilinearly with respect to the seashore in a calm sea. If one jumps on this ship, he will land in the same place of the ship, if one looks at a fish tank full of little fishes, he will note that the surface of the tank's water is flat, and that the movements of the fishes is indistinguishable from those they have in a tank on the mainland, if some drops of water fall into a bucket, they will fall as if the ship were at rest. Therefore, only relying upon these observations and experiments, it will be impossible to distinguish if the ship or the seashore is moving (Galilei, 1967, Day 2, pp. 186-188). This means that the ship is the physical equivalent of an inertial reference frame. It is to point out that this is true only very approximately, also neglecting water's friction, because the sea is not a plane, but a portion of an ellipsoidal surface and because the Earth is subject to many motions. However, for the local movement in which Galileo was interested, Galileo's ship can be reasonably regarded as equivalent to an inertial frame.

Newton, in the fifth corollary to his laws of motion, clearly established what happens when a portion of space is at rest or moves uniformly:

> When bodies are enclosed in a given space, their motions in relation to one another are the same whether the space is at rest or whether it is moving uniformly straight forward without circular motion (Newton, 1999, p. 423).

Newton's space is the equivalent of our inertial reference frames. However, it is both part of the theory and of the universe. In Newton's time, the distinction between a model of the universe and the universe itself was not as clear as it is nowadays. This granted, Newton wondered whether in the universe something which were completely at rest and to which all the other inertial movements could be referred could exist. This is an extremely problematic issue, because Newton was well aware, as the previous corollary and the whole of his treatment shows, that two inertial frames (or portion of space) are indistinguishable as to the physical effects. However, though he being perhaps the greatest genius ever existed, he was a man of the 17th century and he felt the need to anchor his inertial spaces and objects to an absolute inertial object, which he thought to have identified in absolute space, an infinite extended container in which the scene of the universe takes place and that exists independently of events and objects.

The problem dealt with by Newton is crucial from an educational standpoint, as well. To what can the inertia of a body be referred? In other words, with respect to what can it be determined? Descartes referred every movement of the bodies to that of the neighbouring bodies (Descartes 1984). In the celebrate bucket experiment, Newton proved that the inertia of a body cannot be referred to the neighbouring bodies, but he

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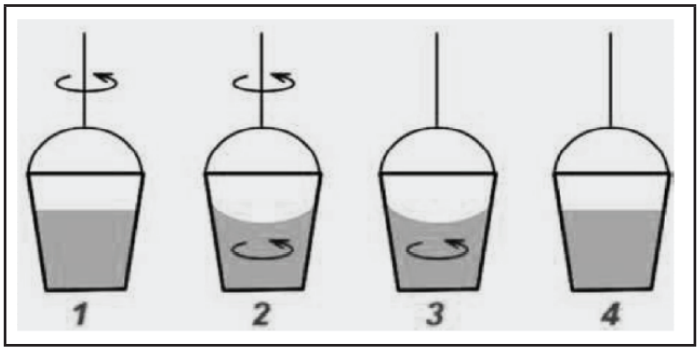
went far beyond, thinking to have deduced the existence of an at rest, infinite, absolute space. Let us see his reasoning (Bussotti 2021):

> Consider a bucket hanging at a rope (Figure 8). Act like this:
> (1) Twist the rope until it is very hard and keep the rope twisted;
> (2) Fill the bucket of water;
> (3) Free the rope. You observe that the bucket begins to rotate, but you will observe no movement in the water (picture 1, Figure 8);
> (4) After a little bit of time, the bucket, because of the friction between the particles of the water and those of the bucket's walls, communicates the movement to the water, which begins to rotate and to ascend along the bucket's walls (picture 2, Figure 8);
> (5) After a while, the bucket stops its movement, but the water continues to rotate (picture 3, Figure 8);
> (6) Finally, both the bucket and the water are at rest (picture 4, Figure 8).

Figure 8
Images Showing Newton's Bucket Experiment



Newton interpreted the rotational movement of the water as due to its inertia. He was perfectly right: during its movement, each particle of water tends to escape along its instantaneous tangent because of the inertia principle. Inertia tendency cannot be actualised because of the bucket's walls. Thence, the particles ascend on the bucket's walls, and the entire water's surface assumes a concave form.

Newton wondered to what the ascending movement of the water, namely its inertia, can be referred. The answer is in the following Table 1.

https://doi.org/10.33225/BalticSTE/2025.47

Table 1.
A Summary of Newton's Bucket Experiment

	Relative motion water/bucket	Water's surface	Inertial forces
1.	Max.	Flat	No
2.	No	Curved	Yes
3.	Max.	Curved	Yes
4.	No	Flat	No

The inertia of water cannot be referred to the bucket, because there are effects of the inertial tendency (the ascent of the water and its curved shape) both in the case in which the relative motion water/bucket is null (phase 2) and in that in which it is maximal (phase 3). You note the lack of inertial tendency both in the case in which the relative motion of water/bucket is maximal (phase 1) and in that in which no relative motion exists (phase 4).

Ergo, Newton concluded that water's inertia cannot depend either on the bucket or, even less, on other objects close to the water. He referred inertia to the absolute space he had previously defined, an infinite and neutral container of the whole universe. In this way, the concept of inertia is, in Newton's view, well founded; it is possible, at least in principle, if not in practice, to distinguish the inertial reference frames from the others: they are the frames which are at rest or move rectilinearly and uniformly in respect to the Absolute Space.

Newton was legitimate to conclude that inertia cannot be referred to neighbouring bodies, but he was not to claim it can be referred to absolute space, because such a space can in no way be distinguished from other inertial spaces, or reference frames in our perspective (Belkind, 2007, Bussotti & Lotti, 2022, pp. 207-235, DiSalle, 1994, DiSalle, 2002, Gould, 1962, Hugett, 2008, Reichenbach, 1957, Rynasiewicz, 1995, Stein, 1967). This notwithstanding, Newton's absolute space was accepted by most physicists until the end of the 19th century, with some remarkable exceptions such as Leibniz, because it offered a sort of psychological sureness that the concept of inertial motion, which is so difficulty to be well defined, but at the same time so crucial, is, after all, based on an existing entity: absolute space. With Mach's work, the situation changed because he had the idea to refer the inertia of a body *B* to the presence of all the other bodies of the universe, so that in a universe with different bodies, the inertia of *B* would have been different. It is interesting to remark that, at the beginning of the 20th century, experiments and observations were conceived to prove if Mach's conception was correct, but no detectable result was obtained (see Bussotti-Lotti, 2022, pp. 225-235). The picture dramatically changed with Einstein's general relativity, although, even nowadays, inertia is not easily explainable. But this is another story which goes far beyond the limit of this research. With this panorama on inertia, we hope to have highlighted important aspects useful from an educational perspective.

https://doi.org/10.33225/BalticSTE/2025.47

Brief considerations on Newton's third law

With regard to the third law of dynamics, namely the action and reaction principle, we restrict ourselves to quoting its enunciation and an explanation given by Newton himself, which offers a clear justification of such a principle. We read:

> To any action, there is always an opposite and equal reaction; in other words, the actions of two bodies upon each other are always equal and always opposite in direction. (Newton, 1999, p. 417).

To explain the nature of this principle, Newton developed the following acute reasoning, which is useful in an educational perspective too (ibidem, pp. 417-430): consider two bodies *A* and *B* which mutually attract. Pose an obstacle between the two bodies. If the third law were not valid and one of the two bodies, for example, *A,* were attracted by *B* more than *B* by *A*, the obstacle could not remain in equilibrium because it is more pressed by *A* than by *B*. The system of two bodies-obstacle will, thence, accelerate from *A* toward *B*. But this contradicts the principle of inertia because a system on which no external action is exerted would move with an accelerated motion. This is a perfect explanation of the reason why the third law is necessary in Newton's physics. It depends on the first law, which, ergo, once again shows its crucial aspect in classical dynamics.

Conclusions and Implications

The main purpose of this research is the attempt to underline the role that history of science and mathematics can play within science and mathematics education. I have been working on this project for more than ten years. Basically, I deal with mathematics education (see Introduction), but I also tried to show the usefulness of using the history of physics in the teaching of this discipline with respect to the concept of inertia and energy (see Introduction). The advantage of the historical approach is that the learners can grasp how some concepts were introduced because of the internal necessities of mathematics and physics. These concepts are, thence, comprehended in their genetic phase and become more understandable for the students who can appreciate the work of genial scholars starting from their problems and their conceptions. On the other hand, I do not consider the historical approach to science education a universal access key. For example, in the teaching of quantum mechanics, it is not appropriate to begin relying upon history because Planck reached the idea of energy quantum by studying the black-body radiation, which is a difficult subject, connected to many nuances and problems of physics at the end of the 19th century. There are more suitable ways to introduce quantum mechanics to learners without dealing with the questions connected to black-body radiation. Therefore, I am not proposing a solution valid once and forever. Nonetheless, the historical approach works well on many occasions. Furthermore, it has the remarkable advantage of realizing an interdisciplinary education, which is one of the main tenets of modern pedagogy. It is important that the teachers know different methods to teach their discipline and that they choose the best, also considering the students who are in front of them. The methods can vary according to the quality and the interests of the pupils.

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